

## 國立臺北教育大學 98 學年度學士班轉學考試

學系 (組)：數學暨資訊教育學系 (數學組)

年 級：三年級

科 目：高等微積分

1. Let  $f: S \rightarrow T$  be a function. If  $A$  and  $B$  are arbitrary subsets of  $S$ , prove that  $f(A \cup B) = f(A) \cup f(B)$ . (10%)

2. Show that the intersection of any two open sets is open in  $\mathbb{R}^2$ . (15%)

3. 利用夾擊定理證明  $\lim_{n \rightarrow \infty} \frac{10^n}{n!} = 0$ . (15%)

4. Deduce  $\frac{d}{dx} \sin x = \cos x$  from the definition of derivative. (10%)

5. Test for convergence ( $p$  and  $q$  denote fixed real numbers).

(a)  $\sum_{n=1}^{\infty} n^3 e^{-n}$ . (10%)

(b)  $\sum_{n=1}^{\infty} \frac{1}{p^n - q^n}$ , where  $0 < q < p$ . (10%)

6. 設  $f(x)$  在  $[0,1]$  上連續，求證：存在  $c \in (0,1)$ ，使得

$$\int_0^1 f(x) dx = f(c) - c^2 + \frac{1}{3}. (15\%)$$

7. Let  $f$  be continuous on  $[a,b]$  and let  $f(x) = 0$  when  $x$  is irrational.

Prove that  $f(x) = 0$  for every  $x$  in  $[a,b]$ . (15%)