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國立臺北教育大學96學年度學士班轉學考試

學系：數學暨資訊教育學系　三年級

科目：高等微積分

1. Let $a_1 = \sqrt{2}$, and let a_n for $n \geq 2$ be defined recursively by the formula $a_{n+1} = \sqrt{2+\sqrt{a_n}}$. Prove that $\lim_{n\to\infty} a_n$ exists. (10%)
2. Evaluate the integral $\int_{-\infty}^{\infty} e^{-x^2}\,dx$. (10%)
3. Prove that: the union of countable collection of finite sets is countable. (10%)
4. State the definition of a Cauchy sequence and prove that every convergent sequence is a Cauchy sequence. (10%)
5. Let $f(x,y) = \begin{cases} \dfrac{xy(x^2-y^2)}{x^2+y^2}, & \text{if } (x,y) \neq (0,0) \\ 0 & , \text{if } (x,y) = (0,0) \end{cases}$, find $\dfrac{\partial^2 f}{\partial x \partial y}(0,0)$ and $\dfrac{\partial^2 f}{\partial y \partial x}(0,0)$. (10%)
6. Complete the following definitions or statements: (5% each)
   (1) Let (M_1,d_1) and (M_2,d_2) be two metric spaces. We say that $f: M_1 \to M_2$ is uniformly continuous if ________________
   (2) Suppose that $f:[a,b] \to R$ is bounded. Then $f:[a,b] \to R$ is integrable if __
   (3) A metric space (M,d) is said to be complete if ________________
   (4) Given a function $f: D \subseteq R \to R$ and a sequence of functions $f_n: D \to R, n = 1,2,\ldots$, the sequence $\{f_n\}$ is said to converge uniformly to f if ________________
7. Prove or disprove that every connected subset of R^n is pathwise connected. (10%)
8. Let (M,d) be a metric space and $f: M \to M$ such that $d(f(x),f(y)) < d(x,y)$ for all $x, y \in M$ and $x \neq y$. Prove or disprove that f has a fixed point. (10%)
9. Find the Taylor series of $f(x,y) = x^3 + y^3 + xy^2$ at the point $(1,2)$. (10%)