國立台北教育大學 96 學年度學士班轉學考試

學系：數學暨資訊教育學系 二年級

科目：代數

1. Suppose that (G, •) is an abelian (commutative) group. State its definition, Please. (15pts)

2. Suppose that relation E is an equivalent relation on non-empty S and

$\overline{a_E} \equiv \{b \in S \mid b \;\; s.t. \;\; a E b\} \overset{def}{=}$ the equivalent class of a $w.r.t$ E.

Show that $\overline{a_E} = \overline{b_E} \Leftrightarrow a \;\; E \;\; b \;\; (i.e: (a,b) \in E) \Leftrightarrow \overline{a_E} \cap \overline{b_E} \neq \phi$ (15pts)

3. Suppose that $(G, *, 1_G)$ is a group and $K \subseteq_S G$. Show

K is normal in G $\Leftrightarrow$ $gK = Kg, \; \forall g \in G$ (10pts)

4. Suppose that G is a simple abelian group. Show that G is cyclic with one or prime order (15pts)

5. The fundamental theorem of cyclic groups says that every subgroup of a cyclic group is cyclic.

List all the subgroup of Z_{30} by applying this theorem. Here, $Z_{30} = \{0,1,2,\ldots,29\}$. (15pts)

6. Let $R[x]$ denote the ring of polynomials with real coefficients

and let $<x^2+1>$ denote the principal ideal generated by x^2+1; that is ,

$<x^2+1> = \{f(x)(x^2+1) \mid f(x) \in R[x]\}$.

Show that $<x^2+1>$ is a maximal ideal in $R[x]$. (15pts)

7. We have known that a polynomial of degree n over a field has at most n zeros counting multiplicity.

Find all complex zeros of x^n+1. (15pts)