# 國立臺北教育大學101學年度學士班轉學考試

學系（組）：數學暨資訊教育學系

年　　級：三年級

科　　目：高等微積分

1. 求證：$\int_0^{\infty} e^{-x^2}dx = \frac{\sqrt{\pi}}{2}$ (10%)

2. Is

$$f(x,y)=\begin{cases} \frac{y^2}{x^2+y^2} & x \neq y \\ 0 & x = y \end{cases}$$

differentiable at $(0,0)$. (10%)

3. State the following two theorems.
   (a) The Bolzano-Weierstrass Theorem (5%)
   (b) The Rolle's Theorem (5%)
   (c) The Mean Value Theorem. (5%)
   (d) The Inverse Function Theorem (5%)

4. If $f$ is differentiable at $a$, show that $f$ is coutinuous at $a$.(10%)

5. $f(x,y) = \frac{1}{x}\sin(xy)$, if $x \neq 0, f(0,y) = y$, determine whether the following limits exits and evaluate those limits that do exits: (1) $\lim_{x\to 0}\lim_{y\to 0} f(x,y)$, (2) $\lim_{y\to 0}\lim_{x\to 0} f(x,y)$, and (3) $\lim_{(x,y)\to(0,0)} f(x,y)$. (15%)

6. .Let f: S→ $\mathbb{R}$ be continuous on an open set S in $\mathbb{R}^n$. Assume that f(p)>0. Prove that there is an n-ball B(p; r) such that f(x)>0 for every x in the ball.(15%)

7. Assume that both $f_n \to f$ and $g_n \to g$ uniformly on S. Prove that $f_n + g_n \to f + g$ uniformly on S. (20%)