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國立臺北教育大學102學年度學士班轉學考試

學系（組）：數學暨資訊教育學系（數學組）

年　　級：大二

科　　目：微積分

1. Define

$$f(x)=\begin{cases} x\sin\dfrac{1}{x}, & x\neq 0,\\ 0, & x=0.\end{cases}$$

Show that the function $f(x)$ is continuous on $\mathbb{R}$ but not differentiable at $x=0$. (15%)

2. Let $\{x_n\}_{n=1}^{\infty}$ be a sequence in $\mathbb{R}$. If $x_n\to\alpha$ as $n\to\infty$, show that

$$\frac{x_1+x_2+\cdots+x_n}{n}\to\alpha \text{ as } n\to\infty. \ (15\%)$$

3. Evaluate the limit

$$\lim_{x\to\infty}\left(1-\frac{3}{x}\right)^{x}. (10\%)$$

4. Use Mean-Value Theorem for Differentiation to prove the **Second Fundamental Theorem of Calculus**: (15%)

Let f be continuous on an interval $[a,b]$ and define $F'(x)=f(x)$ on $[a,b]$. Then $\int_a^b f(x)dx=F(b)-F(a)$.

5. Find the volume of the solid generated when the region enclosed by the given curves is revolved about the x-axis : $y^2 = x,\ y = 1,\ x = 0.$ (15%)

6. Compute the iterated integral

$$\int_0^4 \int_{\sqrt{y}}^2 e^{x^3}\,dxdy\,.\,(15\%)$$

7. Determine whether the series converges or diverges

(a) $\sum_{k=1}^{\infty} k^2 \sin^2 \frac{1}{k}$ (5%)

(b) $\sum_{k=1}^{\infty} \frac{\tan^{-1} k}{1+k^2}$ (5%)

(c) $\sum_{k=1}^{\infty} \frac{\ln k}{e^k}$ (5%)