

國立臺北教育大學 102 學年度學士班轉學考試

學系 (組): 數學暨資訊教育學系 (數學組)

年 級: 大三

科 目: 高等微積分

1. (20%) Define $f_n(x) = \frac{1}{nx}$. Show that $f_n(x) \rightarrow 0$ pointwise but not uniformly on $(0,1)$ as $n \rightarrow \infty$.
2. (20%) Let E be an open interval and f_n be uniformly continuous on E . If $f_n \rightarrow f$ uniformly on E , show that f is also uniformly continuous on E .
3. (20%) Suppose that $A \subseteq B \subseteq \mathbb{R}^n$. Denote \bar{A} by the closure of A and A° by the interior of A . Prove that

$$\bar{A} \subseteq \bar{B} \quad \text{and} \quad A^\circ \subseteq B^\circ.$$

4. (20%) Let $Q \equiv [0,1]^n$ and $\vec{y} = (1,1, \dots, 1)^T \in \mathbb{R}^n$. Show that

$$\int_Q e^{-\vec{x} \cdot \vec{y}} d\vec{x} = \left(\frac{e-1}{e}\right)^n.$$

5. (20%) Show that

$$f(x, y) = \begin{cases} \frac{x^4 + y^4}{(x^2 + y^2)^\alpha}, & (x, y) \neq (0, 0), \\ 0, & (x, y) = (0, 0), \end{cases}$$

is differentiable on \mathbb{R}^2 for all $\alpha < 3/2$.